## COMP3151/COMP9151

## Foundations of Concurrency

Time allowed: 2 hours (9:45-12:00)
Total number of questions: 5
Total number of marks: 45
Textbooks, lecture notes, etc. are not permitted, except for 2 double-sided A4 sheets of hand-written notes.

Calculators may not be used. (Not that they would be of any help.)
Not all questions are worth equal marks.
Answer all questions.
Answers must be written in ink.
You can answer the questions in any order.
You may take this question paper out of the exam.
Write your answers into the answer booklet provided. Use a pencil or the back of the booklet for rough work. Your rough work will not be marked.

## Shared-Variable Concurrency (15 Marks)

## Question 1 (8 marks)

Let $k>1$. Let $A$ be an algorithm which was designed to solve the mutual exclusion problem for 2 processes. The algorithm $B$ for $n=2^{k}$ processes is built up inductively by splitting the $n$ processes into two groups of $\frac{n}{2}$ processes each. In each group the processes compete to enter the critical section using recursively the solution for $\frac{n}{2}$ processes. The winners of each of the two groups use $A$ to determine which one is allowed to enter the top-level critical section.
level 2


Figure 1: The tournament tree for 8 processes. At each level the nodes are numbered from left to right starting from 0 . Thus, each node in the tree is uniquely determined by its level and node number.

Another way to view this idea is to consider the competition between the processes as a knockout tournament, as illustrated in Figure 1. Processes start as leaves in a balanced binary tree. To enter its critical section, each process attempts to progress to the root of the tree, where at each level of the tree it participates in an instance of $A$ with at most one process in its neighbour's sub-tree. The winner at the top level is allowed to enter its critical section. Upon exiting its critical section, the winner traverses the reverse path (from the root to the leaf) executing its post protocol of $A$ at each level. Each instance of $A$ will use a separate set of shared and local variables to avoid interference.

Prove or disprove:
(a) If $A$ satisfies mutual exclusion then so does $B$.
(b) If $A$ satisfies eventual entry then so does $B$.

## Question 2 (7 marks)

Recall the fast mutual exclusion algorithm:

| Algorithm: Fast algorithm for two processes |  |  |  |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { integer gate } 1 \leftarrow 0 \text {, gate } 2 \leftarrow 0 \\ & \text { boolean wantp } \leftarrow \text { false, wantq } \leftarrow \text { false } \end{aligned}$ |  |  |  |
|  | p |  | q |
|  | gate1 $\leftarrow \mathrm{p}$ <br> wantp $\leftarrow$ true |  | gate1 $\leftarrow \mathrm{q}$ <br> wantq $\leftarrow$ true |
| p2: | $\begin{aligned} & \text { if gate } 2 \neq 0 \\ & \text { wantp } \leftarrow \text { false } \\ & \text { goto p1 } \end{aligned}$ |  | if gate $2 \neq 0$ wantq $\leftarrow$ false goto q1 |
| p3: | gate2 $\leftarrow \mathrm{p}$ | q3: | gate2 $\leftarrow$ q |
| p4: | ```if gate1 }\not=\textrm{p wantp}\leftarrow\mathrm{ false await wantq = false``` |  | if gate $1 \neq \mathrm{q}$ <br> wantq $\leftarrow$ false <br> await wantp $=$ false |
| p5: | if gate $2 \neq \mathrm{p}$ goto p 1 else wantp $\leftarrow$ true critical section | q5: | if gate $2 \neq \mathrm{q}$ goto q 1 else wantq $\leftarrow$ true critical section |
| p6: | gate2 $\leftarrow 0$ | q6: | gate2 $\leftarrow 0$ |
| p7: | wantp $\leftarrow$ false | q7: | wantq $\leftarrow$ false |

Does it matter if we change the order of the last two statements in p (lines p 6 and p 7 )?

## Message-Passing Concurrency (30 Marks)

Answers to questions that require programming can be formulated using Ben-Ari's pseudo-code notation, Promela, or (if you must) C with MPI.

## Question 3 (8 marks)

Develop an implementation of a time-server process. The server provides two operations that can be called by client processes: one to get the time of the day and one to delay for a specified interval. In addition, the time server receives periodic "tick" messages from a clock interrupt handler. Also show the client interface to the time server for the time of day and delay operations.

## Question 4 (7 marks)

Develop a solution for the dining philosophers problem under the restriction that a channel must be connected to exactly one sender and one receiver.

## Question 5 (15 marks)

Recall the dining cryptographers' problem. As before we assume that each cryptographer $C_{i}$ has a secret bit $p_{i} \in\{0,1\}$ that is 1 iff $C_{i}$ paid. At most one of the cryptographers paid, but maybe none of them paid but the NSA did.

Consider the following algorithm for a party of $n$ of them. Cryptographer $C_{1}$ secretly flips a coin to generate a result $c \in\{0,1\}$. She then XORs $c$ with $p_{1}$ and passes the result $c \oplus p_{1}$ on to her neighbour $C_{2}$. Every cryptographer $C_{i}$ where $i>1$ waits for input from $C_{i-1}$, XORs that input with $p_{i}$, and passes it on to his neighbour $C_{(i \bmod n)+1}$. When $C_{1}$ receives the bit $b$ from $C_{n}$, she announces "the NSA paid" if $b=c$ and "one of us paid" otherwise.

3 marks Model the algorithm with transition diagrams.
2 marks Formulate a pre- and a postcondition to capture the crucial aspect of the algorithm, namely that the final announcement by $C_{1}$ is truthful.

5 marks Prove validity of the resulting Hoare-triple.
5 marks Consider the case $n=3$. Prove or disprove that $C_{1}$ does not learn which one of $C_{2}$ and $C_{3}$ paid if one of them did.


